Velocity Estimation on the Bayesian Occupancy Filter for Multi-Target Tracking

Manuel Yguel, Christopher Tay, Kamel Mekhnacha, Christian Laugier

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Velocity Estimation on the Bayesian Occupancy Filter for Multi-Target Tracking

Manuel Yguel — Christopher Tay — Kamel Mekhnacha — Christian Laugier

N° 5836

Janvier 2005

Thème NUM

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--5836--FR+ENG

Velocity Estimation on the Bayesian Occupancy Filter for Multi-Target Tracking

Manuel Yguel , Christopher Tay , Kamel Mekhnacha , Christian Laugier

Thème NUM — Systèmes numériques
Projet E-MOTION

Rapport de recherche n° 5836 — Janvier 2005 — 17 pages

Abstract: Reliable and efficient perception in dynamic environments especially in densely cluttered environments is still a challenge today. Most of the systems used for target tracking are based on object models. Such approaches usually fail in complex environments with a variety of different moving obstacles. In this report, we propose a new non object based model to perform tracking in such environments. The approach is called Bayesian Occupancy Filtering (BOF). It basically consists of regular grids where each cell contains information on the distribution on the grid occupancy and velocity. The cells are assumed to be independent from one another to avoid combinatorial explosion. The estimation of cell occupancy and cell velocity are performed in a manner similar to classical filtering, where there is a prediction and estimation stage. Sensory data from different sensors can be fused onto BOF cells and since cells are independent from one another, the notion of physical objects does not exist in this space. Doing so avoids the problem of data association which have to be often resolved in object based tracking systems.

This report describes how the filtering is performed on this grid space in the context of target tracking. The disadvantage of tracking in grid space is when the cells move with a velocity which results in a cell position that does not end in only one single cell. A proposed method of handling such discretisation by staggering the updates of the cell such that cells are updated only when the fit is perfect. This corresponds to the intuitive notion of updating cells with higher velocity at a higher frequency. Such attention focusing enables the reduction of computation burden.

Key-words: Multi-target tracking, bayesian state estimation, occupancy grid

L'Estimation de Velocité dans le Filtre d'Occupation Bayesien pour la Poursuite Multi-Cibles

Résumé : Percevoir de manière efficace et robuste un environnement dynamique, spécialement lorsque de nombreux objets y manoeuvrent en même temps est un des problémes majeur de la robotique et de la percéption en général. La pluspart des systèmes conçus pour la poursuite multi-cibles sont intrinsèquement dépendant de la notion d'objet. Ce type d'approche butte usuellement sur de nombreux problèmes dès que le nombre et le type d'obstacle dans la scène devient important. Dans ce rapport nous proposons un nouveau model, ne s'appuyant pas sur la notion d'objet, pour effectuer du tracking en environnement dynamique. Ce model est une extension du Filtre d'Occupation Bayésien (BOF en anglais). Il est centré autour d'une représentation à base de grille dans les cellules de laquelle sont stockées une distribution de probabilité sur l'occupation de la cellule et sur la vitesse de l'objet occupant la cellule. Pour éviter l'explosion combinatoire, chaque cellule est considérée indépendante des autres. L'estimation de l'occupation et de la vitesse d'une cellule suit une méthode classique de filtrage bayésien avec une étape de prédiction et une étape d'observation. Des mesures provenant de différents cpateurs peuvent être intégrées dans les cellules du BOF et grâce au fait que les cellules sont indépendantes les unes des autres la notion d'objet physique n'existe pas dans cette représentation. Avec une telle représentation, le problème classique de l'association de données entre objets n'a plus de sens ce qui simplifie l'ensemble de la chaîne de traitements tout en la rendant plus robuste.

Ce rapport décrit la manière dont la méthode est implémentée dans l'espace discrétisé pour un problème de poursuite multi-cibles. Les problèmes dans de tels espaces proviennent de la discrétisation notamment lorsque des cellules bougent avec une vitesse telle que leur positions finales ne correspondent pas exactement à des cellules de la grille. Nous proposons une méthode pour gérer ce type de cas et en particulier prendre en compte uniquement les mouvements qui font correspondre parfaitement des cellules de la grille. Cela correspond à l'idée intuitve qui consiste à mettre à jour à haute fréquence les cellules avec une haute vitesse ce qui permet une meilleur répartition de la charge de calcul.

Mots-clés : poursuite multi-cibles, filtrage bayésien, grille d'occupation

Contents

1 Introduction

Perception and reasoning with dynamic environments is pertinent for mobile robotics and constitutes always a major challenge. To work in thoses environments, the mobile robot needs to perceive through its sensors, which measurements are very often uncertain. Dealing with this uncertainty is often treated within the estimation framework. Such an approach enables the mobile robot to model the dynamic environment and follow the evolution of its environment. With an internal representation of the environment, the robot is thus able to perform reasoning and especially predictions in order to accomplish its tasks sucessfully. Systems for tracking the evolution of the envionment has been traditionally a major component in robotics. The industries are now beginning to express their interests in such technologies. One particular example is the application within the automotive industry for adaptive cruise control [4]. One major challenge is to reduce road accidents, thanks to better collision detection systems. The major requirement of such a system is a robust tracking system. Most of the existing target tracking algorithms use object-based representation of the environement. However, these existing techniques require an explicit handling of data association and occlusion problems. In view of these problems, the bayesian occupancy filter (BOF) [5] [4] has been proposed.

The five main motivations in the proposed BOF approach are:

- *Taking uncertainty into account explicitly*, uncertainty which is inherent in any model of a real phenomenon. The uncertainty is manifested as a variant of occupancy grids;
- *Avoiding the "data association problem"* in the sense that the data association is to be handled at a higher level of abstraction. The data association problem is to associate an object o_t at time instant t with o_{t+1} at time instant $t+1$. Current methods for resolving the data association problem often does not perform satisfactorily under complex scenarios *i.e* scenarios involving numerous appearances, disappearances and occlusions of several rapidly manoeuvring targets. The concept of objects is non existant in the BOF and hence avoids the problem of data association in the classical tracking sense;
- *Avoiding the object model problem*, that is avoid to make asumption about the shape or the size of the object. Indeed it is really complex to define what the sensor could measure without a good representation of the object. In particular a big object can cause multiple detections whereas a little object cause just one. In both case there is only one object and that lack of coherence, causes the multiple target tracking systems to, most of the case, work properly with only one kind of target.
- An *Increase in the robustness of the system relative to object occlusions, appearances and disappearances* by the exploitation at any time, of all relevant information on the environment perceived by the vehicle. This information includes the description of occupied and hidden areas (*i.e.* areas of the environment that are temporarily hidden to the sensors by an obstacle.

- *A method that could be implemented later on a dedicated hardware*, in order to both reach high performance and descrease the costs of the final system. That is a method which is as parallel as possible.

We claim that in the BOF approach, the five previous objectives are met by:

- *Uncertainty* taken into account explcitly thanks to the *probabilistic reasoning paradigm*, which is becoming a key paradigm in robotics: various approaches based on this paradigm have already been successfully used to address several robotic problems, such as CAD modelling [11] or map building and localisation (SLAM) [15, 9, 1].

- *The data association problem* is postponned by reasoning on a probabilistic grid representation of the dynamic environment. In such a model, concepts such as *objects* or *tracks* do not exist; they are replaced by more useful properties such as occupancy or risk, which are directly estimated for each cell of the grid using both sensor observations and some prior knowledge. Furthermore, when estimating occupancy probability thanks to an adequate sensor model, the hidden parts of the environments can also be explicitly characterised. Since we consider both the positions and the velocities of the potential obstacles with respect to our vehicle, this grid is 4-dimensional and is called the *Obstacle State Space (OSS)* grid.

- *The object model problem* is non existant because there are only cells in the environment state and that each sensor measurement changes the state of each cell. The fact that different kind of object can produce different kind of measure is handled naturally by the cell space discretization.

- *The dynamicity of the environment and the robustness relatively to objects occlusions* is addressed using a novel two-steps mechanism allowing to take the *sensor observations history and the temporal consistency of the scene* into account. This mechanism estimates, at each time step, the state of the occupancy grid by combining a prediction step (history) and an estimation step (new measurements). This approach is derived from the *Bayes filters* approach [8]; which explains why our filter is called the *Bayesian Occupancy Filter (BOF)*.

- *The Bayesian Occupancy Filter has been designed in order to be highly parallelisable.* So a hardware implementation on a dedicated chip is possible, which will lead to a really efficient way to represent the environment of an automotive vehicle.

1.1 Contributions

Previous experiments based on the BOF techniques relied on the assumption of constant velocity and the problem of velocity estimation in this context has not been addressed. Moreover in previous implementation, there was no link between cells that share position but no velocity. In particular the assumption that it could only be one object with one velocity in each cell was not part of the previous model. In the following article, a representation that has one probability distribution over velocities for each occupancy cell is presented. This model is very similar in its concepts to optical flow, but with occupancy considerations rather than intensity. The general principle for the estimation of occupancy grids will be to include the velocity estimation in the prediction estimation loop of the classical BOF approach. For each grid in the BOF, the set of velocities that brings a set of corresponding cells in the previous time step to the current grid will be considered. The resulting distribution on the velocity of the current grid is updated by conditioning the incoming velocities to the current grid on the observations. A detailed synthesis of the method will be described in chapter 3.

This report brings an amelioration in this aspect and thus presents an integrated approach to performing not only the occupancy states of the grids but the distribution on grid velocity as well. The current paper is organized as follow:

- in chapter 2 multiple target tracking system and occupancy grids are presented. Following by the fundamental concepts of the bayesian occupancy filter.
- in chapter 3 the new filtering equations are widely described in addition with all the algorithm steps.
- in chapter 4 the problem due to space discretization are explained and the new solution we propose are described.

2 Related Work

2.1 Multi-Target Tracking

The estimation of the dynamic characteristics of the traffic participants is basically a *multi-target tracking* problem. The objective is to collect *observations*, *i.e.* sensor data, on one or more *potential obstacles* in the environment of the vehicle, and then to estimate at each time step (and as robustly as possible) the obstacles positions and velocities. Classical approach is to track the different objects independently, by maintaining a list of *tracks*, *i.e.* a list of currently known objects. The main difficulty of multi-target tracking is known as the *Data Association* problem. It includes observation-to-track association and track management problems. The goal of observation-to-track association is to decide whether a new sensor observation corresponds to an existing track or not. Then track management includes deciding whether existing tracks should be maintained or deleted, and whether new tracks should be created. Numerous methods exist to perform data association [2, 7, 14]. The reader is referred to [3] for a complete review of the existing tracking methods with one or more sensors.

Urban traffic scenarios are still a challenge in multi-target tracking area: the traditional data association problem is intractable in situations involving numerous appearances, disappearances and occlusions of a large number of rapidly manoeuvring targets.

In [17], a classical Multiple Hypothesis Tracking technique is used to track moving objects while stationary objects are used for SLAM. Unfortunately, the authors did not explicitly address the problem of the interaction between tracked and stationary objects, *e.g.* when a pedestrian is temporary hidden by a parked car. It is one of the purpose of our approach to solve this problem.

2.2 Grid Representation of the Environment

The *occupancy grids* framework [12, 6] is a classical way to describe the environment of a mobile robot. It has been extensively used for static indoor mapping using a 2-dimensional grid [16]. The goal is to compute from the sensor observations the probability that each cell is full or empty. To avoid a combinatorial explosion of grid configuration, the cell states are estimated as *independent* random variables.

More recently, occupancy grids have been adapted to track multiple moving objects [13]. In this approach, spatio-temporal clustering applied to *temporal maps* is used to perform motion detection and tracking. A major drawback of this work, relatively to the ADAS context, is that a moving object may be lost due to occlusion effects.

2.3 The bayesian occupancy filter

This section presents the standard approach to tracking in the BOF in which additional dimensions is added to the grids where the dimensions represents the velocity. The purpose is mainly to illustrate the differences between the new technique proposed and to provide a foundational comprehension of the bayesian occupancy filter.

2.3.1 Problem addressed and approach

We are now interested in taking into account the sensor observation history, in order to be able to make more robust estimations in changing environments (i.e. in order to be able to process temporary objects occlusions and detection problems). Our approach for solving this problem is to make use of an appropriate Bayesian filtering technique called the *Bayesian Occupancy Filter (BOF)*.

Bayes filters [8] address the general problem of estimating the state sequence x^k, $k \in N$ of a system given by:

$$x^k = f^k(x^{k-1}, u^{k-1}, w^k), \tag{1}$$

where f^k is a possibly nonlinear transition function, u^{k-1} is a "control" variable (*eg* speed or acceleration) for the sensor which allows to estimate its ego-movement between time $k-1$ and time k, and w^k is the process noise. This equation describes a Markov process of order one.

Let z^k be the sensor observation of the system at time k. The objective of the filtering is to recursively estimate x^k from the sensor measurements:

$$z^k = h^k(x^k, v^k). \tag{2}$$

where h^k is a possibly nonlinear function and v^k is the measurement noise. This function models the uncertainty of the measurement z^k of the system's state x^k.

In other words, the goal of the filtering is to recursively estimate the probability distribution $P(X^k \mid Z^k)$, known as the *posterior* distribution. In general, this estimation is done in two stages: *prediction* and *estimation*. The goal of the prediction stage is to compute an *a priori* estimate of the target's state known as the *prior* distribution. The goal of the estimation stage is to compute the *posterior* distribution, using this *a priori* estimate and the current measurement of the sensor.

Exact solutions to this recursive propagation of the posterior density do exist in a restrictive set of cases. In particular, the Kalman filter [10][18] is an optimal solution when the functions f^k and h^k are linear and the noises w^k and v^k are Gaussian. But in general, solutions cannot be determined analytically, and an approximate solution has to be computed.

In this case, the state of the system is given by the occupancy state of each cell of the grid, and the required conditions for being able to apply an exact solution such as the Kalman filter are not alway verified. Moreover, the particular structure of the model (occupancy grid) and the real-time constraint coming from the ADAS application, has to the development of the concept of *Bayesian Occupancy Filter*. This filter consists of estimating the occupancy state in a two-steps, as depicted in fig 1.

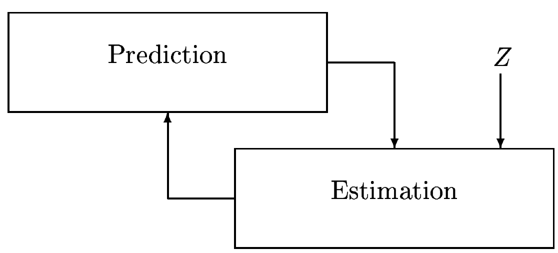

Figure 1: Bayesian Occupancy Filter as a recursive loop.

3 The Bayesian Occupancy Filter With Velocity Estimation

In this chapter, the improvements to the standard BOF approach described in chapter 2 are presented. The main contribution of this approach is a novel method of performing the estimation of velocity and grid occupancy in a single integrated framework. With such a velocity estimation process, it is now possible to focus the computation on the most probable velocites of grids instead of updating the grid occpancy values for every possible velocity in the standard BOF approach. Such an approach is not only more efficient computationally, but provides a more theoretically founded and systematic way of estimating grid velocities.

3.1 Bayesian Model

3.1.1 Probabilistic variable definitions

All the probabilistic variables below is within the context of a single cell c of the grid. This subscript is now ommitted everywhere to maintain simplicity, except where some ambiguity is possible.

One sensor model

- C is an index that identify each 2D cell of the grid.
- A is an index that identify each possible antecedant of the cell c over all the cells in the 2D grid.
- $Z_t \in \mathcal{Z}$. Z_t is the random variable of the sensor measurement relative to the cell c.
- $V \in \mathcal{V} = \{v_1, \ldots, v_n\}$. V is the random variable of the velocities for the cell c and its possible values are discretized in n cases.
- $O, O^{-1} \in \mathcal{O} \equiv \{occ, emp\}$. O represents the random variable of the state of c either "occupied" or "empty". Same as above, O^{-1} is the random variable of the state of an antecedent cell of c through the possible motion of c. For a given velocity $v_k = (v_x, v_y)$ and a given time step δt, it is possible to define an antecedent for $c = (x, y)$ as $c^{-k} = (x - v_x \delta t, y - v_y \delta t)$.

3.1.2 Joint distributions

The following expression is the decomposition of the joint distribution of the all the relevant variables according to bayes' rule.

$$\begin{aligned} & P(C, A, Z, O, O^{-1}, V) \\ & \quad = P(A)P(V|A)P(C|V, A)P(O^{-1}|A)P(O|O^{-1})P(Z|O, V, C) \end{aligned} \tag{3}$$

- $P(A)$ is the distribution over all the possible antecedant of the cell c. It is choosen uniform because the cell is considered reachable from all the antecedants with the same probability.
- $P(V|A)$ is the distribution over all the possible velocities of a certain antecedant of the cell c, its parametric form is an histogram.
- $P(C|V,A)$ is a distribution that explains if c is reachable from $[A = a]$ with the velocity $[V = v]$. It is a dirac with value equal to one if and only if $c_x = a_x + v_x\delta t$ and $c_y = a_y + v_y\delta t$ which follows a dynamic model of constant velocity.
- $P(O^{-1}|A)$ is the conditional distribution over the occupancy of the antecedant. It gives the probability of the possible previous step of the the current cell.
- $P(O|O^{-1})$ is the conditional distribution over the occupancy of the current cell. It depends of the occupancy state of the previous cell. It is defined as a transition matrix: $T = \begin{bmatrix} 1-\epsilon & \epsilon \\ \epsilon & 1-\epsilon \end{bmatrix}$, which allows the system to take in account the fact that the nul acceleration hypothesis is an approximation; in this matrix, ϵ is a parameter representing the probability that the object in c does not follow the nul acceleration model.
- $P(Z|O,V,C)$ is the conditional distribution over the sensor measurement values. It depends of the state of the cell, the velocity of the cell and obviously the position of the cell. Including velocity in the dependances allows to include sensors that could give velocity measurements.

Th aim of the filtering is to estimate the distribution and grid velocity for each grid. The global filtering equation can be obtained by:

$$P(V,O|Z,C) = \frac{\sum_{A,O^{-1}} P(C,A,Z,O,O^{-1},V)}{\sum_{A,O,O^{-1},V} P(C,A,Z,O,O^{-1},V)} \tag{4}$$

Which can be made in three stages when notice that:

$$P(V,O,Z,C) = P(Z|O,V,C)(\sum_{A,O^{-1}} P(A)P(V|A)P(C|V,A)P(O^{-1}|A)P(O|O^{-1})),$$

where the sum is the result of the prediction stage, the other term standing for the observation stage.

3.1.3 Calculus stages:

The global filtering equation (eqn. 4) can actually be separated into three stages. The first being the prediction of the grid occupancy given the velocity:

$$\begin{aligned}\alpha(occ, v_k) &= \sum_{A,O^{-1}} P(A)P(v_k|A)P(C|A)P(O^{-1}|A)P(occ|O^{-1}),\\ \alpha(emp, v_k) &= \sum_{A,O^{-1}} P(A)P(v_k|A)P(C|A)P(O^{-1}|A)P(emp|O^{-1}).\end{aligned} \tag{5}$$

The observation (defined for a certain velocity) can be given by:

$$\begin{aligned}\beta(occ, v_k) &= P(Z|occ, v_k)\alpha(occ, v_k),\\ \beta(emp, v_k) &= P(Z|emp, v_k)\alpha(emp, v_k).\end{aligned}$$

Third the likelihood of a certain velocity is given by:

$$l(v_k) = \beta(occ, v_k) + \beta(emp, v_k).$$

Finally:

$$P(occ, v_k, Z, C) = \frac{\beta(occ, v_k)}{l(v_k)}. \tag{6}$$

The occupancy value in one cell is obtained by the marginalisation sum over the velocities and the velocity distribution by the marginalisation over the occupancy values:

$$P(O|Z, C) = \sum_V P(V, O|Z, C), \tag{7}$$

$$P(V|Z, C) = \sum_O P(V, O|Z, C). \tag{8}$$

What can be surprising is that, a tracking system will only focus on the velocity for occupied cells, that is $P(occ, V)$ and only for cells that are high probability to be occupied.

4 Problem of discretisation and the choice of velocities

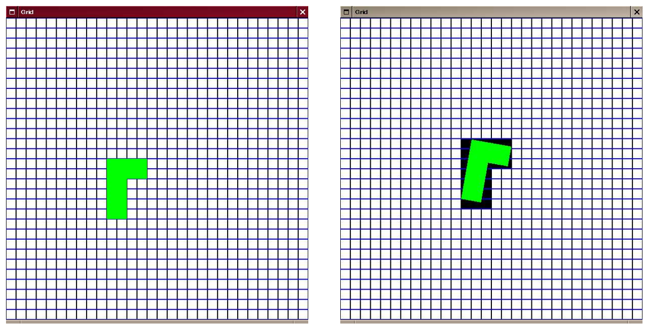

Figure 2: Aliasing problem: the area of an object counted in occupied cell number is not constant for each position of the object in the grid.

The space discretisation in the grid introduces quiet difficult problems, first of all the occupancy of a cell is considered to be the same in the entire cell, and for some arbitrary displacement of one occupied cell, some areas of multiple cells are occupied[1] (fig. 2). The consequence is that, if such a displacement of a cell is allowed, the prediction step introduces an error. To correct this error, observations must be more trusted, which leads to a filter more sensible to false detections. In such a case, the whole quality of the filter dicreases.
Therefore we introduce a new prediction scheme for step one (eq. 5) and two (eq. 6) that take the velocity of a cell into account. The key idea is to choose a possible velocity of a cell such that it corresponds to a displacement of an exact integer number of cells in the grid. It strongly depends from the time step dt of filtering updates. We consider dt as a constant in the rest of the paper. The consequence is that the grid is regular and we choose a cartesian grid with a constant step for each space dimension: dx and dy in 2D.
Thus the set of possible velocities is:

$$\mathcal{V} = \{(\frac{p}{n}\frac{dx}{dt}; \frac{q}{n}.\frac{dy}{dt})|(p,q,n) \in \mathbb{Z}^2 \times \mathbb{N}^\star\}$$

where $\frac{1}{n}$ allows to consider movements of a cell that requires more than one time step to reach totally an other cell. For example, a translation of dx which requiers 2 time steps corresponds to the velocity vector: $(\frac{dx}{2dt}; 0)$ and the corresponding (p,q,n) are $(1,0,2)$. There is a particular case: $p = q = 0$, for which only $n = 1$ is considered.

[1]This problem is known as the aliasing problem

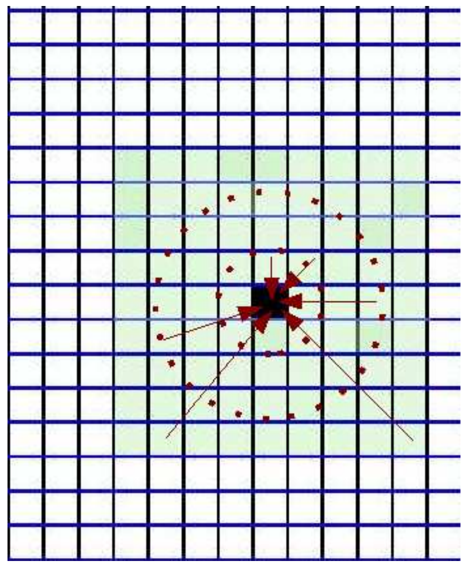

Figure 3: Each considered velocity corresponds to a movement that allows to reach the central cell from any neighbouring cell in at most s time steps.

4.1 Consequences for observation step

The key idea is to propagate and to observe a velocity plan with (p, q, n) velocity only $\frac{1}{n}$ frequency. The consequence is that all velocity plans are not check at each time step, in particular slow velocity plans are observed slowly. Indeed, slow moving objects required less attention than fast moving objects because their location in space change less between 2 time steps. So, this asynchronous updating scheme just reflects an intuitive principle of attention allocation. And this better attention sharing benefits to a better allocation of computing resources.

So we organize the updating process by gathering all velocity plans that share the same frequency. Then for each time step, only groups of velocity plans with a frequency that corresponds to the current time step are updated.

5 Conclusion

This report proposed a novel method of performing the grid occupancy and grid velocity estimation within the Bayesian Occupancy Filter (BOF) framework. In the standard BOF framework, a constant velocity dynamic model for the grids have to be assumed and the problem of estimating the grid velocity has not been addressed.

The report begins with a description of the standard BOF approach given a constant velocity dynamic model. Several experimental results were illustrated, thereby showing the advantages of the such a grid based approach in performing multi target tracking.

Afetr which, a new proposal is described. The velocity of a particular grid is estimated by considering the set of antecedent grids that will bring the set of antecedent grids to the current grid (with its associated velocity given by the constant velocity dynamic model "back projected").

Experimental results for this novel BOF with the estimation of velocity has not been illustrated as the current results are associated with an industrial partner. Detailed and experimental results analysis will be provided in future publications.

References

[1] K.O. Arras, N. Tomatis, and R. Siegwart. Multisensor on-the-fly localization : precision and reliability for applications. *Robotics and Autonomous Systems*, 44:131–143, 2001.

[2] Y. Bar-Shalom and X. Li. *Multitarget Multisensor Tracking : Principles and Techniques*. YBS Publishing, 1995.

[3] S. Blackman and R. Popoli. *Design and Analysis of Modern Tracking Systems*. Artech House, 2000.

[4] C. Coué, Th. Fraichard, P. Bessière, and E. Mazer. Multi-sensor data fusion using bayesian programming: an automotive application. In *Proc. of the IEEE-RSJ Int. Conf. on Intelligent Robots and Systems*, Lausanne, (CH), Octobre 2002.

[5] C. Coué, Th. Fraichard, P. Bessière, and E. Mazer. Using bayesian programming for multi-sensor multi-target tracking in automotive applications. In *Proceedings of IEEE International Conference on Robotics and Automation*, Taipei (TW), septembre 2003.

[6] A. Elfes. Using occupancy grids for mobile robot perception and navigation. *IEEE Computer, Special Issue on Autonomous Intelligent Machines*, Juin 1989.

[7] H. Gauvrit, J.P. Le Cadre, and C Jauffret. A formulation of multitarget tracking as an incomplete data problem. *IEEE Trans. on Aerospace and Electronic Systems*, 33(4), 1997.

[8] A. H. Jazwinsky. *Stochastic Processes and Filtering Theory*. New York : Academic Press, 1970.

[9] L.P. Kaelbling, M.L. Littman, and A.R. Cassandra. Planning and acting in partially observable stochastic domains. *Artificial Intelligence*, 101, 1998.

[10] R.E. Kalman. A new approach to linear filtering and prediction problems. *Journal of basic Engineering*, 35, Mars 1960.

[11] K. Mekhnacha, E. Mazer, and P. Bessière. The design and implementation of a bayesian CAD modeler for robotic applications. *Advanced Robotics*, 15(1):45–70, 2001.

[12] H.P. Moravec. Sensor fusion in certainty grids for mobile robots. *AI Magazine*, 9(2), 1988.

[13] E. Prassler, J. Scholz, and A. Elfes. Tracking multiple moving objects for real-time robot navigation. *Autonomous Robots*, 8(2), 2000.

[14] R.L. Streit and T.E. Luginbuhl. Probabilistic multi-hypothesis tracking. Technical Report 10,428, Naval Undersea Warfare Center Division Newport, 1995.

[15] S. Thrun. Learning metric-topological maps for indoor mobile robot navigation. *Artificial Intelligence*, 99(1), 1998.

[16] S. Thrun. Robotic mapping: A survey. In *Exploring Artificial Intelligence in the New Millenium*. Morgan Kaufmann, 2002.

[17] C-C Wang, C. Thorpes, and S. Thrun. Online simultaneous localization and mapping with detection and tracking of moving objects: Theory and results from a ground vehicle in crowded urban areas. In *Proc of the IEEE Int Conf on Robotics and Automation*, pages 842–849, Taipei (taiwan), September 2003.

[18] G. Welch and G. Bishop. An introduction to the Kalman filter. available at `http://www.cs.unc.edu/~welch/kalman/index.html`.

Unité de recherche INRIA Rhône-Alpes
655, avenue de l'Europe - 38334 Montbonnot Saint-Ismier (France)

Unité de recherche INRIA Futurs : Parc Club Orsay Université - ZAC des Vignes
4, rue Jacques Monod - 91893 ORSAY Cedex (France)
Unité de recherche INRIA Lorraine : LORIA, Technopôle de Nancy-Brabois - Campus scientifique
615, rue du Jardin Botanique - BP 101 - 54602 Villers-lès-Nancy Cedex (France)
Unité de recherche INRIA Rennes : IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex (France)
Unité de recherche INRIA Rocquencourt : Domaine de Voluceau - Rocquencourt - BP 105 - 78153 Le Chesnay Cedex (France)
Unité de recherche INRIA Sophia Antipolis : 2004, route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex (France)

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)
http://www.inria.fr
ISSN 0249-6399